DØ Note 5979-CONF

Determination of the Strong Coupling Constant from the Inclusive Jet Cross Section in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96\,\mathrm{TeV}$

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URL http://www-d0.fnal.gov

(Dated: August 13, 2009)

We determine the strong coupling constant α_s and its running from the p_T dependence of the inclusive jet cross section in $p\bar{p}$ collisions at $\sqrt{s} = 1.96\,\mathrm{TeV}$. The running of the strong coupling constant is shown over the range $50 < p_T < 145\,\mathrm{GeV}$. Using perturbative QCD calculations to order $\mathcal{O}(\alpha_s^3)$ combined with $\mathcal{O}(\alpha_s^4)$ contributions from threshold corrections, we obtain a result of $\alpha_s(M_Z) = 0.1173^{+0.0041}_{-0.0049}$. This is the most precise result from a hadron collider.

Preliminary Results for Summer 2009 Conferences

Asymptotic freedom, the fact that strong forces between quarks and gluons become arbitrarily weak at smaller distances, is a remarkable property of quantum chromodynamics (QCD). This property is reflected by the renormalization group equation (RGE) prediction for the dependence of the strong coupling constant α_s on the renormalization scale μ_r and therefore on the momentum transfer. Experimental tests of asymptotic freedom require precise determinations of $\alpha_s(\mu_r)$ over a large range of momentum transfers. Frequently, α_s has been determined using production rates of hadronic jets in either e^+e^- annihilation or in deep-inelastic ep scattering (DIS). So far there exists only a single α_s result from jet production in hadron-hadron collisions. The CDF collaboration determined α_s from the inclusive jet cross section in $p\bar{p}$ collisions at $\sqrt{s} = 1.8\,\text{TeV}$ as $\alpha_s(M_Z) = 0.1178^{+0.0081}_{-0.0095}(\text{exp.})^{+0.0071}_{-0.0047}(\text{scale}) \pm 0.0059(\text{PDF})$ [1].

In the present analysis we determine α_s and its dependence on the momentum transfer based on the published measurement of the inclusive jet cross section [2] with the DØ detector [3] at the Fermilab Tevatron Collider in $p\bar{p}$ collisions at $\sqrt{s} = 1.96\,\text{TeV}$. The inclusive jet cross section $d^2\sigma_{\text{jet}}/dp_T/d|y|$ was measured using the Run II iterative midpoint cone algorithm [4] with cone radius of 0.7 in rapidity, y, and azimuthal angle. The result comprises 110 data points corrected to the particle level [5] and presented as a function of transverse jet momentum (transverse with respect to the beam direction), p_T, for $p_T > 50\,\text{GeV}$ in six regions of $|y|$ for $0 < |y| < 2.4$.

The ingredients of pQCD calculations in hadron collisions are α_s, perturbative coefficients c_n (in the n-th power of α_s) and the parton distribution functions (PDFs). Conceptually, PDFs depend only on the hadron momentum fraction x carried by the parton and on the factorization scale μ_f. In practice, PDFs are determined from measurements of observables which depend on α_s. Therefore resulting PDF parametrizations depend on the assumption for α_s made in the extraction procedure. For all consistent phenomenology, this implicit α_s dependence must be taken into account consistently. The pQCD prediction for the inclusive jet cross section can therefore be written as

$$\sigma_{\text{pert}}(\alpha_s) = \left(\sum_n \alpha_s^n c_n \right) \otimes f_1(\alpha_s) \otimes f_2(\alpha_s)\,, \tag{1}$$

where the sum runs over all powers n of α_s which contribute to the calculation ($n = 2, 3, 4$ in this analysis, see below). The $f_{1,2}$ are the PDFs of the initial state hadrons and the "$\otimes$" sign denotes the convolution over the momentum fractions x_1, x_2 of the hadrons. Since the RGE uniquely relates the value of $\alpha_s(\mu_r)$ at any scale μ_r to the value of $\alpha_s(M_Z)$, all equations can be expressed in terms of $\alpha_s(M_Z)$. The total theory prediction for inclusive jet production is given by the pQCD result (1) multiplied by a correction factor for non-perturbative effects

$$\sigma_{\text{theory}}(\alpha_s(M_Z)) = \sigma_{\text{pert}}(\alpha_s(M_Z)) \cdot c_{\text{non-pert}}\,. \tag{2}$$

The latter includes corrections due to hadronization and underlying event which have been estimated in Ref. [2] using PYTHIA [6] with CTEQ6.5 PDFs [7], tune QW [8], and $\alpha_s(M_Z) = 0.118$. The perturbative results are the sum of a full calculation to $\mathcal{O}(\alpha_s^3)$ (NLO), combined with the $\mathcal{O}(\alpha_s^4)$ (2-loop) terms from threshold corrections [9]. Adding the 2-loop threshold corrections leads to a significant reduction in the μ_r and μ_f dependence of the calculation. The theory calculations are done in the $\overline{\text{MS}}$ scheme [10] for five active quark flavors for gluon splitting using the next-to-next-to-leading logarithmic (3-loop) approximation of the RGE [11, 12]. The PDFs are taken from the MSTW2008 next-to-next-to-leading order (NNLO) parametrizations [13, 14] and μ_r and μ_f are set to $\mu_{r,f} = p_T$. The calculations are using fastNLO [15] based on NLOJET++ [16, 17] and code from the authors of Ref. [9].

In this analysis, the value of α_s is determined from sets of inclusive jet cross section data points by minimizing the function χ^2 using MINUIT [18]. Where appropriate, the $\alpha_s(M_Z)$ result will be evolved to the scale p_T using the 3-loop solution of the RGE, providing a result for $\alpha_s(p_T)$. All correlated experimental and theoretical uncertainties are treated in the Hessian approach [19], except for the $\mu_{r,f}$ dependence (see below). The central $\alpha_s(M_Z)$ result is obtained by minimizing χ^2 with respect to $\alpha_s(M_Z)$ and the nuisance parameters for the correlated uncertainties. By scanning χ^2 as a function of $\alpha_s(M_Z)$, the uncertainties are obtained from the $\alpha_s(M_Z)$ values for which χ^2 is increased by one with respect to the minimum value.

To determine α_s according to this procedure, knowledge of $\sigma_{\text{pert}}(\alpha_s(M_Z))$ is required as a continuous function of $\alpha_s(M_Z)$, over an $\alpha_s(M_Z)$ range which covers the possible fit results and their uncertainties. This can be achieved based on a series of PDFs obtained under the same conditions but for different values of $\alpha_s(M_Z)$ using interpolation in $\alpha_s(M_Z)$. Some recent PDF analyses have explored this aspect and their results are documented for different values of $\alpha_s(M_Z)$. The MSTW2008 NLO and NNLO PDF parametrizations [13, 14] are presented for 21 $\alpha_s(M_Z)$ values in the range $0.110 - 0.130$ in steps of 0.001 and the CTEQ6.6 results [20] are available for five values of $\alpha_s(M_Z) = 0.112, 0.114, 0.118, 0.122, 0.125$. Due to the wide range in $\alpha_s(M_Z)$ covered by the MSTW2008 PDFs and the fine and equidistant spacing in $\alpha_s(M_Z)$, we use cubic spline interpolation to obtain a smooth parametrization for the $\alpha_s(M_Z)$ dependence of the cross section for $0.111 \leq \alpha_s(M_Z) \leq 0.129$. This range is sufficient to cover our central values and the uncertainties. The MSTW2008 analysis includes data sets that have not yet been included in other global PDF analyses (DIS jet data from HERA and recent CCFR/NuTeV dimuon data) and the results are available in NNLO accuracy which is adequate to be used when including the $\mathcal{O}(\alpha_s^4)$ contributions from threshold corrections in

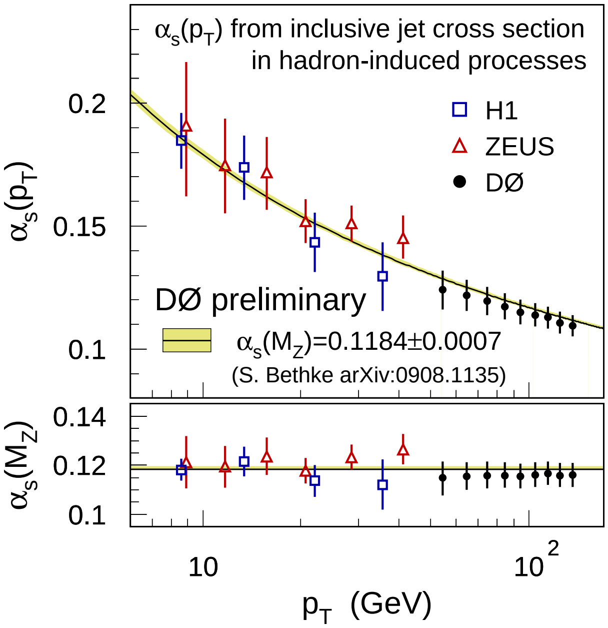

FIG. 1: The running of $\alpha_s(p_T)$ (top). The results are evolved to $\mu_r = M_Z$ (bottom). The DØ results are based on 22 selected data points. For comparison, HERA DIS jet data have been included and also the RGE prediction for the world average value and its uncertainty (line and band). All data points are shown with their total uncertainties.

the cross section calculation. The CTEQ6.6 PDF parametrizations are available up to NLO, for five $\alpha_s(M_Z)$ values, and for a more limited range in $\alpha_s(M_Z)$. Therefore the MSTW2008 PDFs are used to obtain the main results for this analysis while the CTEQ6.6 PDFs are used for comparison.

Care must be taken in phenomenological analyses if the observable under study was already used to provide unique constraints on the PDFs as this introduces correlations of experimental and PDF uncertainties, and it may affect the sensitivity to possible new physics signals. Both aspects are relevant in this α_s determination since the DØ inclusive jet data under study had been included in the MSTW2008 PDF analysis. Since the correlation of experimental and PDF uncertainties is not documented, it can not be taken into account when using the PDFs to extract $\alpha_s(M_Z)$ from the jet data. As a consequence, we must avoid using those jet cross section data points which had provided unique PDF constraints. While the quark PDFs are constrained by precision structure function data, the only direct source of information on the high x gluon PDF comes currently from Tevatron inclusive jet data. The impact of Tevatron jet data on the gluon density is documented in Ref. [13] in Figs. 51-53. Fig. 51 shows that excluding the Tevatron jet data starts to affect the gluon density at $x > 0.2 - 0.3$, while for $x \lesssim 0.25$ the difference in the gluon density is less than 5%. Fig. 53 shows that $x < 0.3$ is the region in which the gluon results for MSTW2008 and CTEQ6.6 are very close. We conclude that for momentum fractions $x < 0.2 - 0.3$ the Tevatron jet data do not have a major impact on the gluon density, and therefore we can neglect correlations between PDF and experimental uncertainties for these data. Based on this constraint we select below those inclusive jet data points from which we extract α_s.

The Tevatron jet data (which access p_T above 500 GeV) are probing momentum transfers at which α_s has not yet been probed in other experiments. Therefore we can not rule out deviations in the running of α_s at large momentum due to possible new physics contributions to the RGE. Since such modifications of the RGE are not taken into account in the PDF determinations, these effects would effectively be absorbed into the PDFs. By construction, using such PDFs to extract α_s could seemingly confirm the RGE expectations, even in the presence of new physics contributions to the RGE. For a consistent α_s determination we would therefore exclude high p_T data in the region where the RGE has not yet been successfully tested which is the region of $p_T \gtrsim 200$ GeV [21]. However, those data are already removed by the restriction to $x < 0.2 - 0.3$, so no additional requirement is needed to account for this.

In $2 \to 2$ processes, given the information of the rapidities and p_T of the two jets one can compute the momentum fractions x_1 and x_2 carried by the initial partons. The inclusive jet cross section at given p_T and $|y|$ is, however, integrating over all additional jets in an event, so the second jet rapidity and therefore the full event kinematics, including x_1 and x_2, are not known. The value of the larger momentum fraction $x_{max} = \max(x_1, x_2)$ can be computed only under an assumption for the rapidity of the unobserved jet. For each inclusive jet $(p_T, |y|)$ bin we define the variable $\tilde{x} = x_T \cdot (e^{|y|} + 1)/2$ where $x_T = 2p_T/\sqrt{s}$, and p_T is the bin center and $|y|$ is the lower boundary of the $|y|$

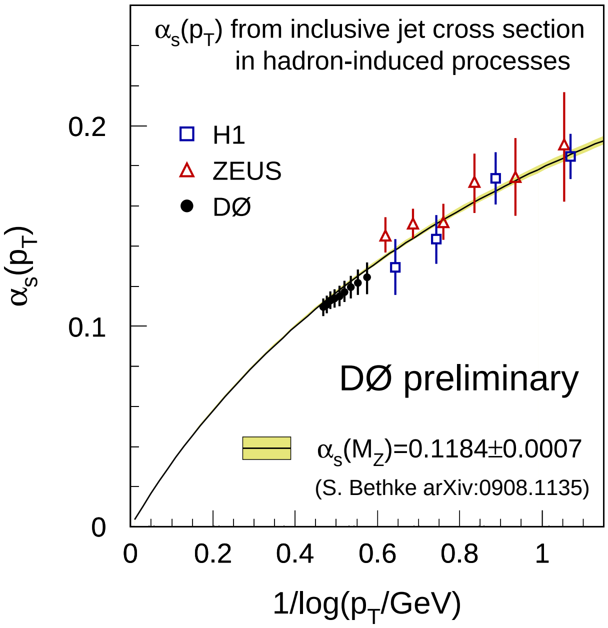

FIG. 2: The running of $\alpha_s(p_T)$ as a function of $1/\log(p_T/\mathrm{GeV})$. For comparison, HERA DIS jet data have been included and also the RGE prediction for the world average value and its uncertainty (line and band). All data points are shown with their total uncertainties.

bin. This variable $\tilde{x}$ corresponds to x_{max} for the case that the unobserved jet was produced at $y = 0$. In the pQCD calculation, for a given inclusive jet $(p_T, |y|)$ bin the distribution of $x_{\max} = \max(x_1, x_2)$ always has a peak plus a tail towards high x values. Although the variable $\tilde{x}$ does not represent the peak position of the x_{max} distribution, it is correlated with the distribution. The requirement $\tilde{x} < 0.15$ removes all data points for which more than half of the cross section is produced at $x_{max} \gtrsim 0.25$. This leaves 22 (out of 110) data points for the α_s analysis with $p_T < 145\,\mathrm{GeV}$ for $0 < |y| < 0.4$, $p_T < 120\,\mathrm{GeV}$ for $0.4 < |y| < 0.8$, $p_T < 90\,\mathrm{GeV}$ for $0.8 < |y| < 1.2$, and $p_T < 70\,\mathrm{GeV}$ for $1.2 < |y| < 1.6$. Although this selection criterion is well-motivated, the specific choices of the variable $\tilde{x}$ and the requirement $\tilde{x} < 0.15$ are somewhat arbitrary. We have therefore studied variations of the selection requirement in the range $\tilde{x} < 0.10 - 0.17$ and other choices for the definition of $\tilde{x}$ (for example assuming that the unobserved jet has $y_2 = \pm|y|$), and, we find that the α_s results are stable within 1%. We conclude that the choice of $\tilde{x} < 0.15$ restricts the jet data to those points which receive no significant contributions from $x_{\max} > 0.25$. For these data points experimental and PDF uncertainties can be assumed to be uncorrelated.

The following uncertainties are considered in the α_s determinations. The uncorrelated experimental uncertainties and all 23 sources of correlated experimental uncertainties as documented in Refs. [2, 22]. The non-perturbative corrections are divided into hadronization and underlying event effects. The uncertainty for each is taken to be half the size of the corresponding effect. PDF uncertainties are computed using the twenty 68% C.L. uncertainty eigenvectors as provided by MSTW2008 [13]. The uncertainties in the pQCD calculation due to uncalculated higher order contributions are estimated from the $\mu_{r,f}$ dependence of the calculations when varying the scales in the range $0.5 < \mu_{r,f}/p_T < 2$. In the kinematic region under study, variations of μ_r and μ_f have positively correlated effects on the jet cross sections. A correlated variation of both scales is therefore a conservative estimate of the corresponding uncertainty. Since the $\mu_{r,f}$ uncertainties can not be treated as gaussian, these are not included in the Hessian χ^2 definition. Following Refs. [23, 24] the α_s fits are repeated for different choices ($\mu_{r,f} = 0.5p_T$ and $\mu_{r,f} = 2p_T$) and the differences to the central result (obtained for $\mu_{r,f} = p_T$) are taken to be the corresponding uncertainties for the $\alpha_s(M_Z)$. Those are added in quadrature to the other uncertainties to obtain the total uncertainty.

Data points from different $|y|$ regions with similar p_T are grouped to determine the results for $\alpha_s(M_Z)$ and $\alpha_s(p_T)$ The results are shown in Fig. 1 as nine $\alpha_s(p_T)$ (top) and $\alpha_s(M_Z)$ values (bottom) in the range $50 < p_T < 145\,\mathrm{GeV}$ with their total uncertainties which are largely correlated between the points. Also included are results at lower p_T from inclusive jet cross sections in DIS from the HERA experiments H1 [23] and ZEUS [24] and the 3-loop RGE prediction for the world average value of $\alpha_s(M_Z) = 0.1185 \pm 0.0007$ [?]. Our $\alpha_s(p_T)$ results are consistent with the energy dependence as predicted by the RGE and extend the HERA results towards higher p_T. The same $\alpha_s(p_T)$ results are also shown in Fig. 2, here as a function of $1/\log(p_T/\mathrm{GeV})$ to demonstrate the asymptotic freedom for $p_T \to \infty$.

Finally, $\alpha_s(M_Z)$ is determined by combining all 22 data points with a result of $\alpha_s(M_Z) = 0.1173^{+0.0041}_{-0.0049}$. The contributions from single uncertainty sources are listed in Table I.

TABLE I: α_s result and the uncertainty contributions

result	uncertainty contributions				
$\alpha_s(M_Z)$	exp. uncorrel.	exp. correl.	non-pert.	PDF	scale $\mu_{r,f}$
0.1173	$^{+0.0001}_{-0.0001}$	$^{+0.0034}_{-0.0029}$	$^{+0.0010}_{-0.0010}$	$^{+0.0012}_{-0.0011}$	$^{+0.0021}_{-0.0029}$

Varying the size of the uncertainties of the non-perturbative corrections between a factor of 0.5 and 2 changes the central value by less than 0.4% and does not effect the uncertainty of the $\alpha_s(M_Z)$ result. Replacing the MSTW2008 NNLO PDFs by the CTEQ6.6 PDFs changes the central result by only +0.5% which is much less than the PDF uncertainty. Excluding the 2-loop contributions from threshold corrections and using pure NLO pQCD (together with MSTW2008 NLO PDFs and the 2-loop RGE) gives a result of $\alpha_s(M_Z) = 0.1202^{+0.0072}_{-0.0059}$. The small increase in the central value is a result of the missing $\mathcal{O}(\alpha_s^4)$ contributions which are compensated by a corresponding increase in α_s. The difference to the central result is well within the scale uncertainty of the NLO result. The increased uncertainty is mainly caused by the increased $\mu_{r,f}$ dependence, but also by the larger PDF uncertainty at NLO.

In summary, we have determined the strong coupling constant from the inclusive jet cross section using theory prediction in NLO plus 2-loop threshold corrections. The $\alpha_s(p_T)$ results support the energy dependence predicted by the renormalization group equation. The combined result from 22 selected data points is $\alpha_s(M_Z) = 0.1173^{+0.0041}_{-0.0049}$. This is the most precise α_s result from a hadron collider.

We thank the staffs at Fermilab and collaborating institutions, and acknowledge support from the DOE and NSF (USA); CEA and CNRS/IN2P3 (France); FASI, Rosatom and RFBR (Russia); CNPq, FAPERJ, FAPESP and FUNDUNESP (Brazil); DAE and DST (India); Colciencias (Colombia); CONACyT (Mexico); KRF and KOSEF (Korea); CONICET and UBACyT (Argentina); FOM (The Netherlands); STFC and the Royal Society (United Kingdom); MSMT and GACR (Czech Republic); CRC Program, CFI, NSERC and WestGrid Project (Canada); BMBF and DFG (Germany); SFI (Ireland); The Swedish Research Council (Sweden); CAS and CNSF (China); and the Alexander von Humboldt Foundation (Germany).

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